1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics Part-2, Class XI)
Module Name/Title	Unit 7, Module 5, Modulus of Rigidity Chapter 9, Mechanical Properties of Solids
Module Id	keph_20905_eContent
Pre-requisites	Modulus of elasticity, stress, strain, bulk modulus of elasticity
Objectives	After going through this module the learners will be able to : • Understand Shear stress and shear strain • Derive an expression for Modulus of rigidity • Study the effect of load on depression of a suitably clamped meter scale load at i) its ends ii) in the middle • Appreciate some effects of shear strain in real life • Know about Poisson's ratio • Calculate elastic energy
Keywords	Shear, height of mountains, shear strain, modulus of rigidity, poisson's ratio, bending of beams

2. Development Team

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1. UNIT SYLLABUS

UNIT 7: PROPERTIES OF B ULK MATTER: 24 periods

Syllabus

Chapter–9: Mechanical Properties of Solids:

Elastic behaviour, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear, modulus of rigidity, Poisson's ratio, elastic energy.

Chapter–10: Mechanical Properties of Fluids:

Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes). Effect of gravity on fluid pressure. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its applications. Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise

Chapter–11: Thermal Properties of Matter:

Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity; Cp, Cv - calorimetry; change of state - latent heat capacity. Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wien's displacement Law, Stefan's law, Greenhouse effect.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 17 MODULES

Module 1	• Forces between atoms and molecules making up the bulk matter • Reasons to believe that intermolecular and interatomic forces exist • Overview of unit • State of matter • Study of a few selected properties of matter • Study of elastic behaviour of solids • Stationary fluid property: pressure and viscosity • Stationary liquid property: surface tension • Properties of Flowing fluids • Effect of heat on matter
Module 2	• Idea of deformation by external force • Elastic nature of materials • Elastic behaviour • Plastic behaviour • Tensile stress • Longitudinal Stress and longitudinal strain • Relation between stress and strain • Hooke's law • Young's modulus of elasticity 'Y'
Module 3	• Searle's apparatus • Experiment to determine Young's modulus of the material of a wire in the laboratory • What do we learn from the experiment?
Module 4	• Volumetric strain • Volumetric stress • Hydraulic stress • Bulk modulus K • Fish, aquatic life on seabed ,deep sea diver suits and submarines
Module 5	• Shear strain • Shear stress • Modulus of Rigidity G • Poisson's ratio • Elastic energy • To study the effect of load on depression of a suitably clamped meter scale loaded at i)its ends ii)in the middle • Height of sand heaps , height of mountains
Module 6	• Fluids-liquids and gases • Stationary and flowing fluids • Pressure due to a fluid column

	• Pressure exerted by solid , liquids and gases • Direction of Pressure exerted by solids, liquids and gases
Module 7	• Viscosity- coefficient of viscosity • Stokes' Law • Terminal velocity • Examples • Determine the coefficient of viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body in the laboratory
Module 8	• Streamline and turbulent flow • Critical velocity • Reynolds number • Obtaining the Reynolds number formula using method of dimensions • Need for Reynolds number and factors effecting its value • Equation of continuity for fluid flow • Examples
Module 9	• Bernoulli's theorem • To observe the decrease in pressure with increase in velocity of a fluid • Magnus effect • Applications of Bernoulli's theorem • Examples • Doppler test for blockage in arteries
Module 10	• Liquid surface • Surface energy • Surface tension defined through force and through energy • Angle of contact • Measuring surface tension
Module 11	• Effects of surface tension in daily life • Excess pressure across a curved liquid surface • Application of surface tension to drops, bubbles • Capillarity • Determination of surface tension of water by capillary rise method in the laboratory • To study the effect of detergent on surface tension of water through observations on capillary rise.
Module 12	• Thermal properties of matter • Heat • Temperature • Thermometers

Module 13	• Thermal expansion • To observe and explain the effect of heating on a bi-metallic strip • Practical applications of bimetallic strips • Expansion of solids, liquids and gases • To note the change in the level of liquid in a container on heating and to interpret the results • Anomalous expansion of water
Module 14	• Rise in temperature • Heat capacity of a body • Specific heat capacity of a material • Calorimetry • To determine specific heat capacity of a given solid material by the method of mixtures • Heat capacities of a gas have a large range • Specific heat at constant volume C_V • Specific heat capacity at constant pressure C_P
Module 15	• Change of state • To observe change of state and plot a cooling curve for molten wax. • Melting point, Regelation, Evaporation, boiling point, sublimation • Triple point of water • Latent heat of fusion • Latent heat of vaporisation • Calorimetry and determination of specific latent heat capacity
Module 16	• Heat Transfer • Conduction, convection, radiation • Coefficient of thermal conductivity • Convection
Module 17	• Black body • Black body radiation • Wien's displacement law • Stefan's law • Newton's law of cooling, • To study the temperature, time relation for a hot body by plotting its cooling curve • To study the factors affecting the rate of loss of heat of a liquid • Greenhouse effect

MODULE 5

3. WORDS YOU MUST KNOW

Elasticity: The property of materials by virtue of which they regain their original shape and size after an external deforming force is removed.

Plasticity: The property of materials by virtue of which they do not regain their original shape and size after an external deforming force is removed.

Deforming force: An external force tending to change length, volume or shape of any body. Deforming forces can act on bodies in any state of matter.

Elastic limit: The extent of external deforming force a body can stand and show the property of elasticity, beyond elastic limit the matter shows property of plasticity.

Strain: The measure of deformity due to external deforming force is called strain.

Longitudinal strain = change in length /original length

Volumetric strain = change in volume / original volume

Stress: The internal force per unit area developed inside the material tending to bring the body back to its original shape and size.

Longitudinal stress = deforming force / area within elastic limit

Volumetric stress: = hydrostatic pressure

Hooke's law: Within elastic limit stress is proportional to strain.

Modulus of elasticity: The ratio of stress and strain is called modulus of elasticity.

Young's modulus Y = longitudinal stress/ longitudinal strain

Bulk modulus K = volumetric stress / volumetric strain

4. INTRODUCTION

Have you ever tried to bend or break a wooden stick but failed because it was too thick? Then you try to support it one end against a wall and step over it to break it. Why does it break now?
An ice-cream stick can easily be broken when applied force to bend it.
How do we wring washed clothes to remove excess water?

How do we make a rope or a wick for oil lamp?


https://cdn.pixabay.com/photo/2016/10/01/22/36/rope-1708542_960_720.jpg

Why do heavy leaves of a plant always fall with a curve?


https://www.savetheredwoods.org/wp-content/uploads/rGrants_LimmGrantPhoto.jpg
https://gospelworkers.files.wordpress.com/2010/01/img_3221.jpg

The seat of the chair is temporarily deformed when we sit on it. Why?

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A small bird does not depress the branch as much as a monkey. Why?

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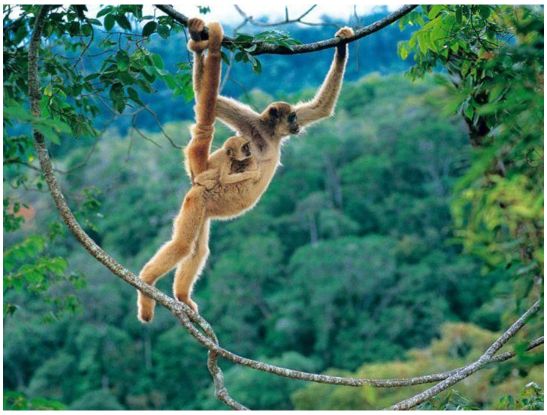


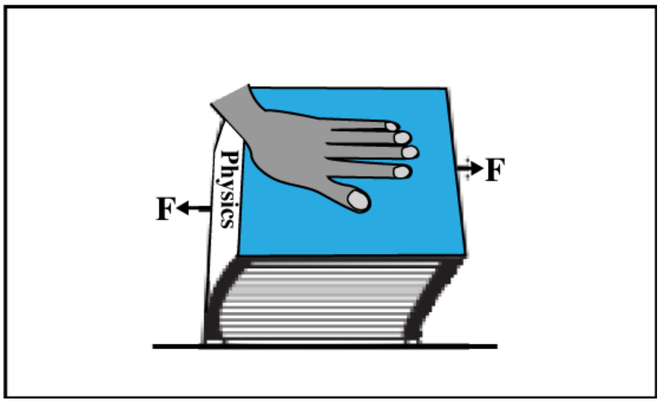
In this module we will discuss another type of force in order to explain the deformities produced in materials around us.

While normal stress and strain are the result of forces being applied perpendicular to the surface of the object, there is another type of stress in which the force is applied tangential or parallel to the object's surface i.e. **shear stress and strain.**

https://i.pinimg.com/736x/45/af/0c/45af0c47209f9659b6fafbfe6709e798--rainforest-animals-amazon-rainforest.jpg

5. SHEAR STRESS

Consider a fat book placed on the table, if we push the top of the book sideways with our palm, it would be deformed.



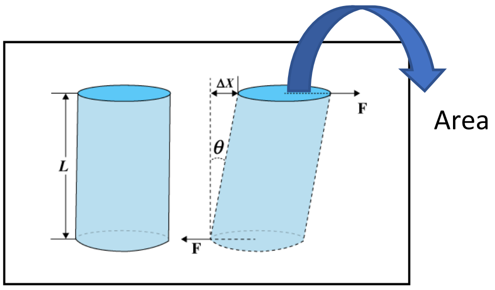
In the above case, the force is applied parallel to the top surface to push it against the table top, thus creating sideways stress or shear in the book.

Due to this tangential force, a cuboid (book) can becomes a parallelogram.

Shear stress, developed inside the book, is equal to the tangential force per unit area of the surface on which the tangential force is applied. The tangential force must be within elastic limit

$$shear\ stress = \frac{tangential\ force}{Area}$$

The area here is the area of the surface on which the tangential force acts. In this case it is the area of the book cover.



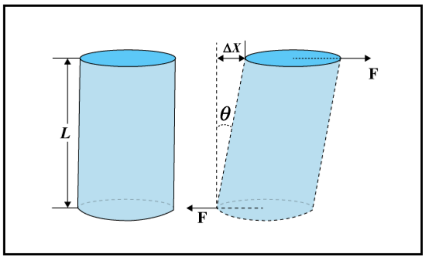
If two equal and opposite deforming forces are applied parallel to the cross-sectional area of the cylinder, there is relative displacement between the opposite faces of the cylinder. The restoring force per unit area developed due to the applied tangential force is known as tangential or shearing stress.

When you cut a piece of card board with a scissor, you apply a force parallel to the cross-section of the cardboard and thus, creating shear stress which breaks it into two.
Now you may think about the shear force acting in each of the situations shown in the pictures. **The scissors are also called shears.**

6. SHEAR STRAIN

Due to the shear stress, the strain produced in the body is shearing strain.
Shearing strain = tangential displacement/ length

The length here refers to the vertical length (with respect to the base) on which tangential force is applied.



As a result of applied tangential force, there is a relative displacement Δx between opposite faces of the cylinder as shown. The strain so produced is known as shearing strain and it is defined as the ratio of relative displacement of the faces Δx to the length of the cylinder L.

Shearing strain = $\Delta x/L = \tan\theta$,
where θ is the angular displacement of the cylinder from the vertical (original position of the cylinder).
Usually θ is very small, tan θ is nearly equal to angle θ, (if θ = 10°, for example, there is only 1% difference between θ and tan θ).
Or
The angle of deformation θ is then termed as the shear strain.
Shear strain is measured in radians and hence is non–dimensional i.e. it has no unit.

7. MODULUS OF RIGIDITY G

According to Hooke's law stress is proportional to strain. Like young's modulus and bulk modulus, the ratio of shear stress and strain a material is undergoing is called a **shear modulus.**

It tells us how a material resists deformation when a tangential force is applied on it. i.e. if two identically shaped different materials (say cubes), one made of aluminium and other made of iron are subjected to same amount of stress, which one would deform more?
If, it is an object made of aluminium and the value of their shear modulus would represent the materials ability to resist deformation due to tangential force.

$$shear\ modulus\ (G) = \frac{shear\ stress}{shear\ strain}$$

Shear modulus is also called as modulus of rigidity (denoted by η or G).

$G = (F/A)/(\Delta x/L) \quad = (F \times L)/(A \times \Delta x)$

Similarly, $G = (F/A)/\theta = F/(A \times \theta)$

The shearing stress σ_s can also be expressed as $\sigma_s = G \times \theta$

SI unit of shear modulus is N m^{-2} or Pa.
The shear moduli of a few common materials are given in Table:

Shear moduli *(G)* of some common materials

Material	G (10^9 Nm^{-2} or GPa)
Aluminium	25
Brass	36
Copper	42
Glass	23
Iron	70
Lead	5.6
Nickel	77
Steel	84
Tungsten	150
Wood	10

It can be seen that shear modulus (or modulus of rigidity) is generally less than Young's modulus. NCERT

- http://philschatz.com/physics-book/contents/m42081.html#import-auto-id1165298671576

Elastic Moduli[1]

Material	Young's modulus (tension-compression) *Y* (10^9 N/m^2)	Shear modulus *S*(10^9 N/m^2)	Bulk modulus *B* (10^9 N/m^2)
Aluminium	70	25	75
Bone - tension	16	80	8
Bone - compression	9		
Brass	90	35	75
Brick	15		
Concrete	20		
Glass	70	20	30
Granite	45	20	45
Hair (human)	10		

From the above mentioned link, we can see that Shear moduli are less than Young's moduli for most materials.

EXAMPLE

The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25GPa. What is the vertical deflection of this face?

SOLUTION:

the edge of aluminium cube =L = 10cm = 0.1m
Mass attached to the cube = m= 100 kg
Shear modulus (η) of aluminium = 25 G Pa = 25×10^{9} N/m^{2}

We have to Find: vertical deflection of the face of cube = ΔL

Shear modulus (η) = $\frac{F/A}{\Delta L/L}$

Force applied by the mass (F) = mg = 100×9.8 = 980N

Area of face of cube (A) = $(0.1)^2 = 0.01\text{m}^2$

Hence,

$$\Delta L = \frac{FL}{A\eta} = \frac{980\times0.1}{0.01\times25\times109}$$

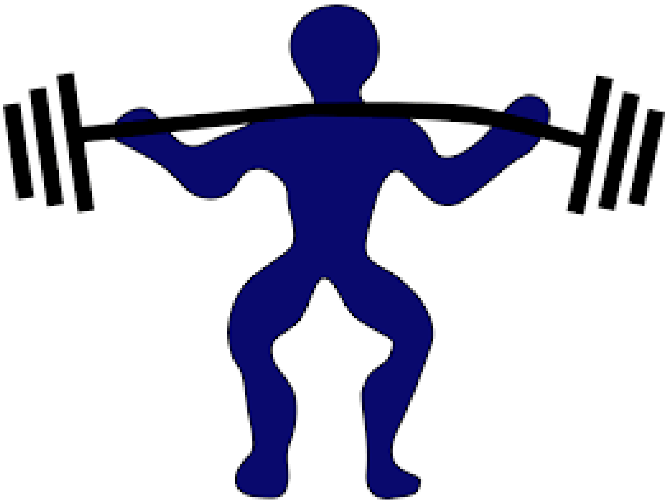
$\Delta L = 3.92\times10^{-7}$m

Bone is a remarkable exception**. Its shear modulus is not only greater than its Young's modulus, but it is as large as that of steel.**

This is why bones are so rigid.

It is due to this that humans and vertebrates have the ability to manipulate their bodies to perform incredible tasks.

The spinal column (consisting of 26 vertebral segments separated by discs) provides the main support for the head and upper part of the body. The spinal column has normal curvature for stability, but this curvature can be increased, leading to increased shearing forces on the lower vertebrae. These higher shear forces increase the risk of back injury through ruptured discs.



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The shear moduli for concrete and brick are very small;

Concrete used in buildings can withstand compression, as in pillars and arches, but is very poor against shear, as might be encountered in heavily loaded floors or during earthquakes.

Modern structures were made possible by the use of steel and steel-reinforced concrete. Liquids and gases have shear moduli near zero, because they flow in response to shearing forces.

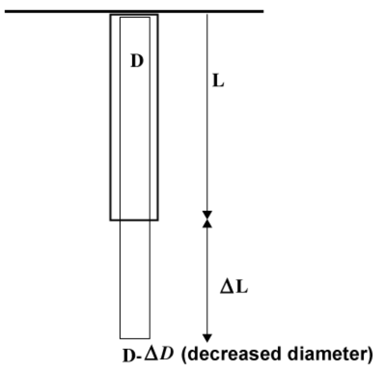
https://c.pxhere.com/photos/89/4b/blackandwhite_bw_monochrome_station_shop_germany_blackwhite_bahnhof-240947.jpg!d

8. POISSON'S RATIO

In general, a deforming force in one direction can produce strains in other directions also. The proportionality between stress and strain in such situations cannot be described by just one elastic constant.

For example, for a wire under longitudinal strain, the lateral dimensions, diameter- (radius of cross section) will undergo a small change, which is described by another elastic constant of the material called **Poisson ratio**.

When a metallic wire is loaded, its length increases but its area of cross-section decreases. The strain produced in the direction of applied force is called longitudinal strain and that produced along the diameter (perpendicular to the length) is called lateral strain.



Within elastic limit, the ratio of lateral strain to longitudinal strain is called Poisson ratio.

$$\sigma = \frac{lateral\ strain}{longitudinal\ strain} = \frac{-\Delta D/D}{\Delta L/L}$$

(The negative sign indicates that diameter decreases when length increases).

Poisson's ratio has no unit and is dimensionless.

9. ELASTIC ENERGY

When elastic material in a catapult is stretched, energy is stored in it as potential energy, and when the catapult is fired, this energy is converted into the kinetic energy of the body released.

When a wire/thread is stretched, restoring force comes into play which opposes the change in it. Work has to be done against the restoring forces in stretching the wire in the form of elastic potential energy.

Let the wire be of unstretched length l and let a force F produce an extension Δl..

Assuming, the elastic limit of the wire has not been exceeded so that no energy is lost in the form of heat.

The work done by force F causing a displacement S is given by, W= F S

But in this case the force varies from 0 at the start, to F at the end when the wire is stretched by an amount Δl.

Therefore, work done during stretching = average force $\times$ extension = ½ F Δl
This work done is stored in the form of elastic potential energy: **U = ½ F Δl**
Multiplying and dividing the above expression by volume: A l

$$U = \frac{1}{2} \times \frac{F \, X \, \Delta l \, X \, A \, l}{A \, l}$$

$$\mathbf{U = \frac{1}{2}\, stress \times strain \times volume\ of\ the\ wire}$$

Energy density is defined as energy per unit volume

Energy density, u = U/Volume

Hence,

$$\mathbf{u = \frac{1}{2} \times Young's\ Modulus \times strain^2}$$

10. TO STUDY THE EFFECT OF LOAD ON DEPRESSION OF A SUITABLY CLAMPED METER SCALE LOAD AT I) ITS ENDS II) IN THE MIDDLE

Before we study the effect of load on depression, let us understand the effect of load on the bridges.

A bridge has to be designed such that it can withstand the load of the flowing traffic, the force of winds and its own weight.

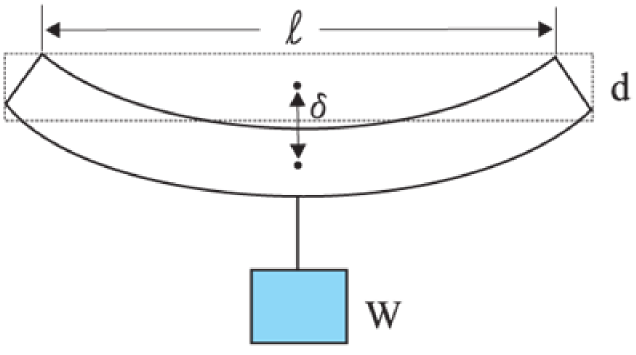
https://upload.wikimedia.org/wikipedia/commons/4/40/HK_Wan_Chai_%E5%B0%9A%E5%8C%AF_The_Gloucester_view_Marsh_Road_bridge_flyover_March_2016_DSC.JPG

Similarly, in the design of buildings, use of beams and columns is very common. In both the cases, the overcoming of the problem of bending of beam under a load is of prime importance.

https://upload.wikimedia.org/wikipedia/commons/9/92/East_Hills_Voyager_Point_footbridge_1.JPG

The beam should not bend too much or break.
Let us consider the case of a beam loaded at the centre and supported near its ends as shown.
A bar (if you consider it as a cuboid) of
Length, l
Breadth, b and
depth /width /height, d
When loaded at the centre by a load, W
sags by an amount given by:

$\delta = \frac{Wl^3}{4bYd^3}$ (we are not deriving this expression here)



A beam supported at the ends and loaded at the center.

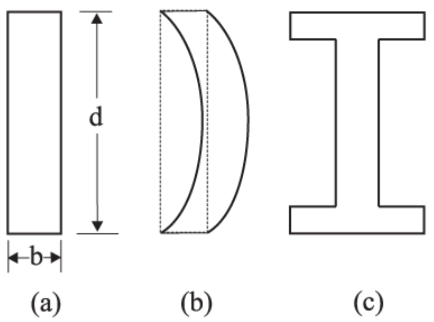
We see that to reduce the bending for a given load, one should use a material with a large Young's modulus Y.

For a given material, increasing the depth d rather than the breadth b is more effective in reducing the bending,
Since, δ is proportional to d^{-3} and only to b^{-1}
Of course, the length l of the span should be as small as possible.

But on increasing the depth, unless the load is exactly at the right place (difficult to arrange in a bridge with moving traffic), the deep bar may bend as shown below. This is called **buckling**.

https://upload.wikimedia.org/wikipedia/en/8/85/Kochi_Metro_U_Girders_at_Kalamassery.jpg

To avoid this, a common compromise is the cross-sectional shape shown of I shown. This section provides a large loadbearing surface and enough depth to prevent bending.
This shape reduces the weight of the beam without sacrificing the strength and hence reduces the cost.

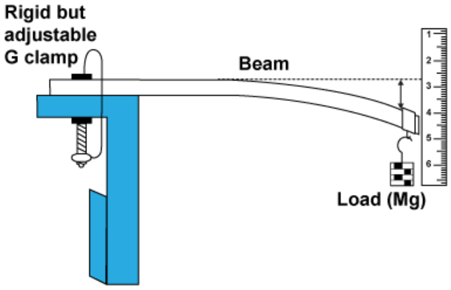


Different cross-sectional shapes of a beam. (a) Rectangular section of a bar; (b) A thin bar and how it can buckle; (c) Commoly used section for a load bearing bar.

TO STUDY THE EFFECT OF LOAD ON DEPRESSION OF A SUITABLY CLAMPED METER SCALE LOAD AT I) ITS ENDS II) IN THE MIDDLE

Apparatus:

Metre scale (**or a thick wooden strip of about 1 m length**), thread, slotted weights with hanger (10g or 20 g each), another graduated scale to be used to measure depression, a pin, cellotape and clamp.



Experimental set up to study depression of metre scale (used as cantilever) with load suspended at free end of the cantilever

Theory:

The depression 'y' of a cantilever of length 'L' clamped at one end and loaded at the free end with a load M (weight Mg) is given by relation

$$y = \frac{MgL^3}{3Y\left(\frac{bd^3}{12}\right)}$$

where L, b and d are length, width and thickness of the rectangular cantilever respectively and Y is the modulus of elasticity of the material of the rod.

OR

$$y = \frac{4MgL^3}{Ybd^3}$$

The readings of depression 'y' of the cantilever, in this case with variation of load suspended at the other end, are taken. The variation of depression with load is expected to be linear.

Procedure:

- Clamp the metre scale firmly to the edge of the table. As shown in Figure, ensure that the length and breadth of the scale are in horizontal plane and 90 cm of the length of the scale is projected out. Fix a pin with a tape at the free end of the metre scale along its length to act as a pointer.

- Fix a graduated scale vertically near the free end of the clamped metre scale and note its least count. Ensure that the pointed end of the pin is just above the graduation marks of the scale but do not touch it.
- Read the pointer 'p' when metre scale cantilever is without any load.
- Suspend a hanger of known mass for keeping slotted weights to depress the free end of the cantilever.
- Read the pointer on vertical scale and record the observation.
- Keep on adding 20 g masses to the hanger and record the reading of the pointer every time it stops vibrating.
- After taking 6-7 observations with increasing load, gradually remove the slotted weights one by one and record the reading while unloading.
- Plot a graph between the depression and the load.

Observations:

Length of the cantilever L = ... cm
Width of the metre scale cantilever b = ... cm
Thickness of the metre scale d = ... cm
Reading of the free end of the cantilever with no load, l_0= E

Effect of load on depression of cantilever

S. No.	Load M (g)	Reading of free end of cantilever			Depression $y = l_m - l_o$
		l_1 (cm) when load is increasing	l_2 (cm) when load is decreasing	Mean $l_m = \frac{l_1 + l_2}{2}$ (cm)	
1					
2					
3					
4					
5					
6					
7					

Result: you may calculate the result
The depression 'y' is directly proportional to the load M.

Precautions:

- The beam should be rigidly clamped at one end.
- Loading and unloading of the slotted weights should be done carefully without disturbing the position of the hanger on the beam.
- The vertical scale should be adjusted close to the pointer in such a way that the pointer moves along it freely.

Sources of Error

- The scale should not be loaded beyond its elastic limit. (This can be easily checked by comparing the zero load reading after removing the maximum suspended load with that taken at the beginning of the experiment).
- There should be no vibratory motion of the beam when reading is recorded.
- While noting down the observation, the eye should be normal to the tip of the pin and the graduated scale.
- Observations should be repeated while removing masses.

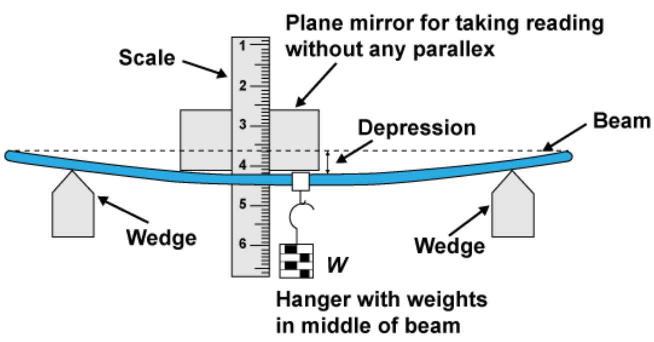
B. BENDING OF METRE SCALE LOADED IN THE MIDDLE

APPARATUS AND MATERIAL REQUIRED Metre scale, two wedges to rest the ends of the metre scale, thread, slotted weights 200 g each, hanger for slotted weights, a graduated scale with a stand to hold the scale vertical, a plane mirror, a pointer and plasticise.

Description of the apparatus

A horizontal metre scale is held on two wedges, a hanger is provided at the middle of the metre scale for applying load.

A pointer is fixed at the mid-point to measure the depression. A graduated (least count 1 mm) scale with a plane mirror strip attached to it is held in vertical position in a stand behind the horizontal metre scale.



Experimental set up to study depression, i.e., sag of a beam with load in the middle

Theory:

Let a beam be loaded at the centre and supported near its ends as shown in Figure, A bar of length 'L', breadth 'b' and thickness 'd' when loaded at the centre by a load 'W' sags by an amount given by

$$Y = \frac{4MgL^3}{Ybd^3}$$

where 'Y' is the Young's modulus of the material of the rod/ beam, W, the load (= mg), where 'm' is the mass of the hanger with weights. The depression 'y' is directly proportional to the load.

Procedure:

- Place the metre scale on two wedges with (5–10 cm) length projecting out on either side. Metre scale supported at both ends is like a beam.
- Tie a loop of thread in the middle of the load such that a hanger to support slotted weights each of 200 g can be suspended on it. Ensure that the thread is tied tightly with the rod and does not slip.
- Place a graduated scale (with least count 0.1 cm) vertically in a stand at the centre of the metre scale used as beam. To facilitate readings the vertical scale should be kept on the far side of the metre scale. Fix a pin to the hanger such that its pointed end is close to the edge of the vertical scale which has graduation marks on it.
- Suspend the hanger of mass 200 g and record the position of the pointer fixed to the hanger. The mirror strip on the vertical scale should be used to remove any parallax.
- Keep on adding 200 g slotted masses to the hanger and record the readings of the pointer each time.
- Take about six observations.
- Now, remove masses of 200g one by one recording the position of the pointer each time while unloading.
- Calculate the depression for the load M gram and hence depression per unit load.
- Plot a graph between the values of depression y against corresponding values of load and interpret the result.

Observations:

Width of the beam, b =
Thickness of the beam, d =
Length of the beam between the wedges, L =

Depression of the beam for different loads:-

S. No.	Load M (g)	Reading of the centre of cantilever			Depression for load M (g), y (cm)	Depression per unit load y/M (cm/g)	Mean y/M (cm/g)
		Load increasing r_1' (cm)	Load decreasing r_2' (cm)	Mean reading $r = \frac{r_1' + r_2'}{2}$ (cm)			
1	0			r_0	0		
2	200			r_1	$r_1 - r_0$		
3	400			r_2	$r_2 - r_0$		
4							
5							
6							

Result:

The depression of the metre scale at its middle is ... mm/g. The depression 'y' is directly proportional to the load M.

Sources of error:

- The rod should not be loaded beyond elastic limit.
- There should be no vibratory motion of the rod when reading is recorded.
- While taking readings, the eye should be normal to tip of the pointer and the metre scale.
- The beam should be of uniform thickness and density throughout its length.
- The masses used must have standard value as engraved on them.

Precautions:

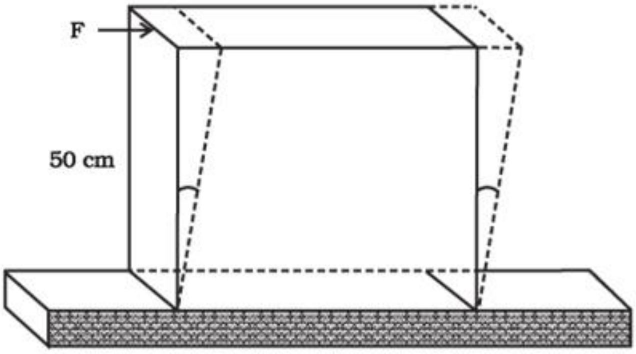
- The beam should be symmetrical on the knife edges.
- Loading and unloading of the slotted weights should be done carefully without disturbing the centre point.
- Mirror strip used to eliminate parallax error should not disturb the experimental set up.

EXAMPLE:

A square lead slab of side 50 cm and thickness 10 cm is subject to a shearing force (on its narrow face) of 9.0 × 10^4 N. The lower edge is riveted to the floor. How much will the upper edge be displaced?

SOLUTION:

The lead slab is fixed and the force is applied parallel to the narrow face as shown.



The area of the face parallel to which this force is applied is A = 50cm×10cm= 0.5 m × 0.1 m = 0.05 m^2

Therefore, the stress applied is = $(9.4 \times 10^4 \text{ N}/0.05 \text{ m}^2) = 1.8 \times 10^6 \text{ N m}^{-2}$

We know that shearing strain = (Δx/L) = Stress /G.

Therefore the displacement: $\Delta x = (\text{Stress} \times L)/G = (1.8 \times 10^6 \text{ N m}^{-2} \times 0.5\text{m}) / (5.6 \times 10^9 \text{ N m}^{-2}) = 1.6 \times 10^{-4} \text{ m} = 0.16 \text{ mm}$

The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.

A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain?

A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 108 N m–2, what is the maximum load the cable can support ?

A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm2. Calculate the elongation of the wire when the mass is at the lowest point of its path.

11. EXPLANATION FOR SOME REAL LIFE SITUATIONS

Bridges are declared unsafe after long use.
Bridges are made of iron, concrete and other materials. They are under continuous stress and strain. Due to continuous use, the material undergoes elastic fatigue and may develop permanent strains.

Metallic parts of machinery get permanently deformed
If materials are subjected to deforming forces beyond elastic limit

Ropes used in cranes to lift heavy weights are thick and are made up of a number of wires

To understand this, consider the following example.

Suppose we need to lift a concrete slab of 100 tons or 10^5 kg.
Elastic limit of steel is $30 \times 10^7 Nm^{-2}$
So the maximum stress the rope can stand is $30 \times 10^7 Nm^{-2}$
Let the radius of the rope be r
Stress $= \frac{load}{area} = \frac{Mg}{\pi r^2}$

$$r^2 = \frac{Mg}{\pi \times stress} = \frac{10^5 \times 10 \times 7}{22 \times 3 \times 10^8} = \frac{7}{6600}$$

Or

$$r = \sqrt{\frac{7}{6600}} = 0.03256m = 3.256\ cm$$

Radius of the rope should be 3 cm, the diameter would be 6 cm and it would be a rigid rod . In actual practise much thicker ropes are used and are made up of thinner wires braided to make them flexible.

Hollow pipe is stronger than a solid rod
Chairs are made of pipes; it is not only to reduce the material being used, the torsional twist is also lesser.

HEIGHT OF SAND HEAPS, HEIGHT OF MOUNTAINS

https://cdn.pixabay.com/photo/2015/06/08/14/52/mountain-801757_960_720.jpg

https://upload.wikimedia.org/wikipedia/commons/3/3a/Tannin_heap.jpeg

Salt poured on a plate heaps up. Why?
Why does a heap of sand or a hill have a pyramidal shape?

The answer to the question why the maximum height of a mountain on earth is ~10 km can also be provided by considering the elastic properties of rocks. A mountain base is not under uniform compression and this provides some shearing stress to the rocks under which they can flow. The stress due to all the material on the top should be less than the critical shearing stress at which the rocks flow.

At the bottom of a mountain of height h, the force per unit area due to the weight of the mountain is $h \rho g$ where ρ is the density of the material of the mountain and g is the acceleration due to gravity.
The material at the bottom experiences this force in the vertical direction, and the sides of the mountain are free.

Therefore, this is not a case of pressure or bulk compression. There is a shear component, approximately $h \rho g$ itself. Now the elastic limit for a typical rock is 30×10^{7} N m^{-2}.

Equating this to $h \rho g$, with $\rho = 3 \times 10^{3}$ kg m^{-3} gives $h \rho g = 30 \times 10^{7}$ N m^{-2} .
Or
$h = 30 \times 10^{7}$ N $m^{-2}/(3 \times 10^{3}$ kg $m^{-3} \times 10$ m $s^{-2}) = 10$ km which is more than the height of Mt. Everest!

Think about this
From the table shown below, we can recall the three types of stress, their corresponding strain, the states of matter in which they are applicable etc.

Stress, strain and various elastic moduli

Type of stress	Stress	Strain	Change in		Elastic modulus	Name of modulus	State of Mater
			shape	volume			
Tensile or compressive	Two equal and opposite forces perpendicular to opposite faces ($\sigma = F/A$)	Elongation or compression parallel to force direction ($\Delta L/L$) (longitudinal strain)	Yes	No	$Y = (F \times L)/(A \times \Delta L)$	Young's modulus	Solid
Shearing	Two equal and opposite forces parallel to oppoiste surfaces [forces in each case such that total force and total torque on the body vanishes ($\sigma_s = F/A$)	Pure shear, θ	Yes	No	$G = (F \times \theta)/A$	Shear modulus	Solid
Hydraulic	Forces perpendicular everywhere to the surface, force per unit area (pressure) same everywhere.	Volume change (compression or elongation ($\Delta V/V$)	No	Yes	$B = -p/(\Delta V/V)$	Bulk modulus	Solid, liquid and gas

Would change in temperature effect elasticity?
What kind of stress occurs when we walk, bite or sleep?
Why are vehicle tyres made of rubber?

12. SUMMARY

In this module we have learnt

- **Shear strain as the angle through which a vertical face of a body is displaced when tangential force is applied on it**
- **Shear stress is the tangential force per unit area developed within the body to restore its shape**
- **Shear modulus of rigidity, G = shear stress /shear strain**
- **Poisson's ratio = lateral strain/longitudinal strain**
- **Elastic potential energy can be calculated**
- **We can study the bending of rods and bars when loaded at one end while other end is fixed or bar loaded at the centre.**